Autoregressive Ranking

Bridging the Gap Between Dual and Cross Encoders

Ben Rozonoyer talk at GDM, February 10, 2026

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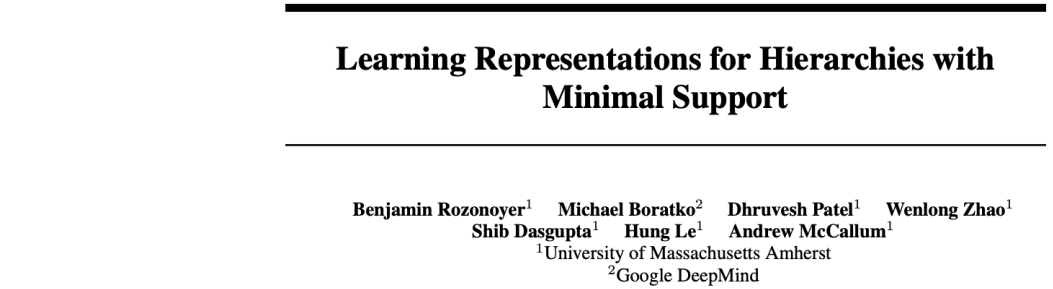
Collaboration with **Chong You**, **Michael Boratko**, **Himanshu Jain**, **Nilesh Gupta**, **Srinadh Bhojanapalli**, **Andrew McCallum** and **Felix Yu**

About me

- PhD candidate (4th) at Information Extraction and Synthesis Lab
- Advised by Andrew McCallum
- Current research focus — discrete diffusion models
- Internships:
  - Transformer-based authorship recognition (Summer 2023)
  - LLM agents for debugging (Summer 2024)
  - Accelerating MoE inference (Summer 2025)
  - Generative Ranking (Spring 2025)



Outline for This Talk

- Introduction
- The Capacity of Autoregressive LLMs for Ranking
  - Ranking via Dual Encoders
  - Ranking via Pointwise Autoregressive Rankers
- A Rank-Aware Loss Function
  - Item-Level Reweighting
  - Prefix Tree for Rank-Aware Token-Level Supervision
- Experiments
  - WordNet
  - ESCI

Introduction

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Prevailing IR paradigm is multistage pipeline:

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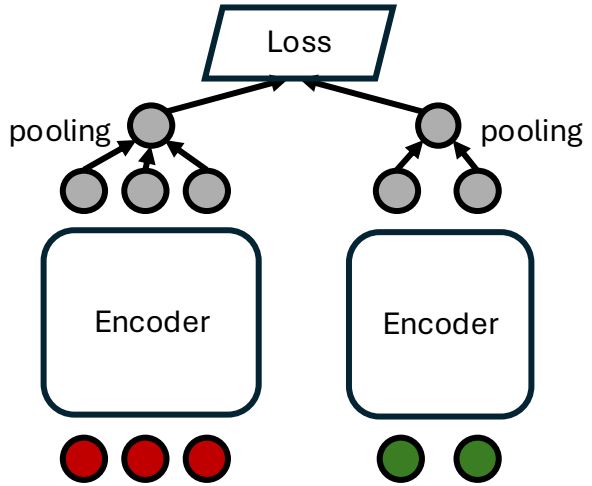

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1. **Retrieval** *(vector similarity)*
   Dual Encoders (DEs)
   Approximate Nearest Neighbors (ANN)

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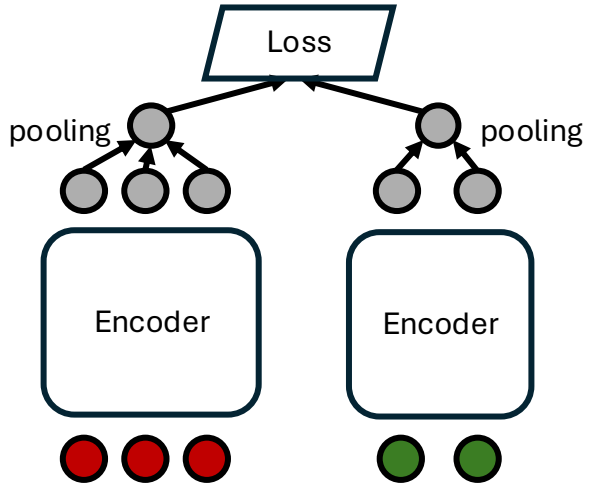
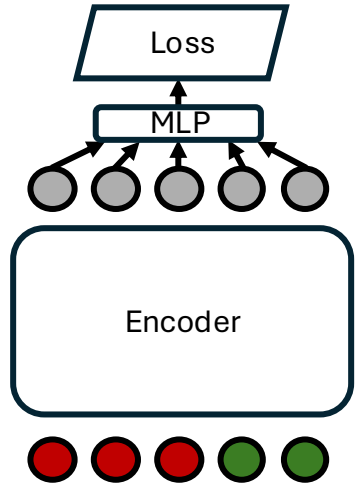
2. **Reranking** *(prohibitive for first-stage retrieval)*
   Cross Encoders (CEs)





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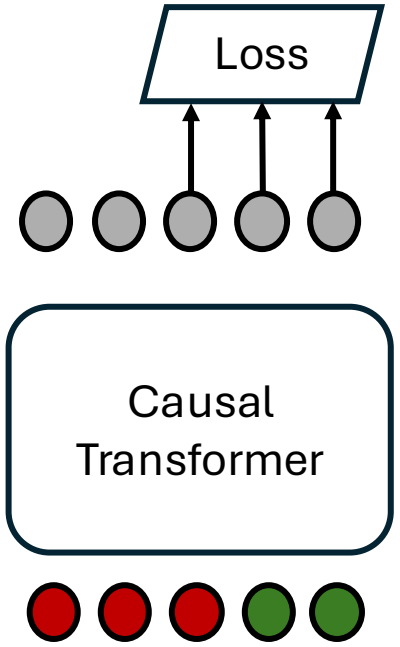

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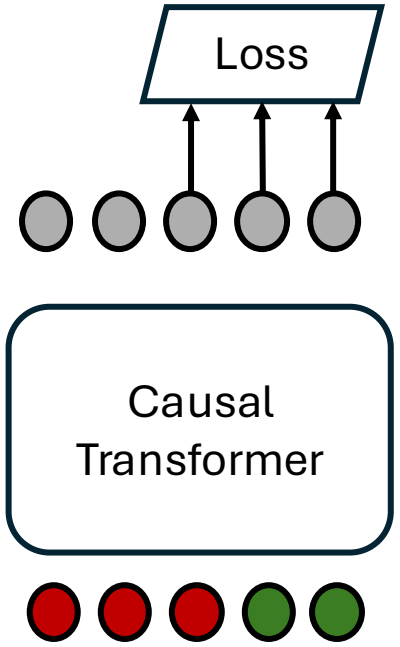

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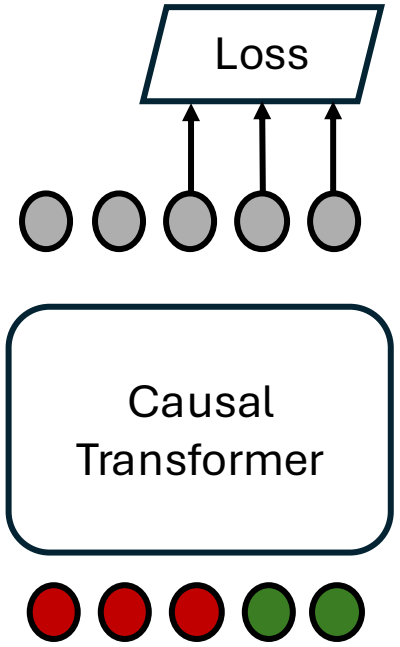
- *More efficient than CEs*
- *More expressive than DEs (empirical evidence)*



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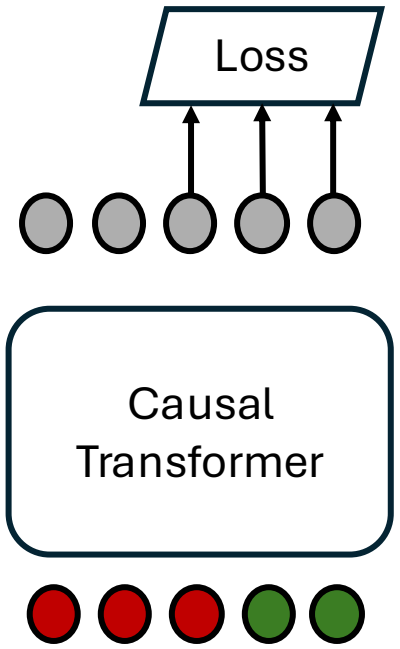
Many LLM rankers are currently trained with NTP

- *rank-agnostic!*



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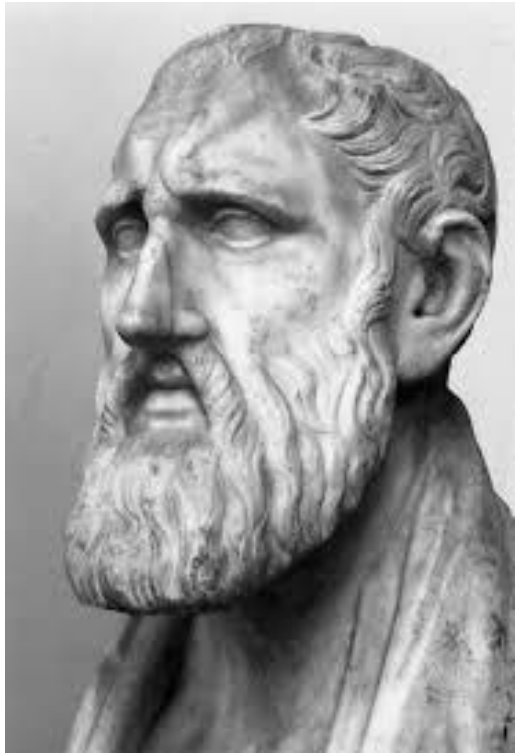

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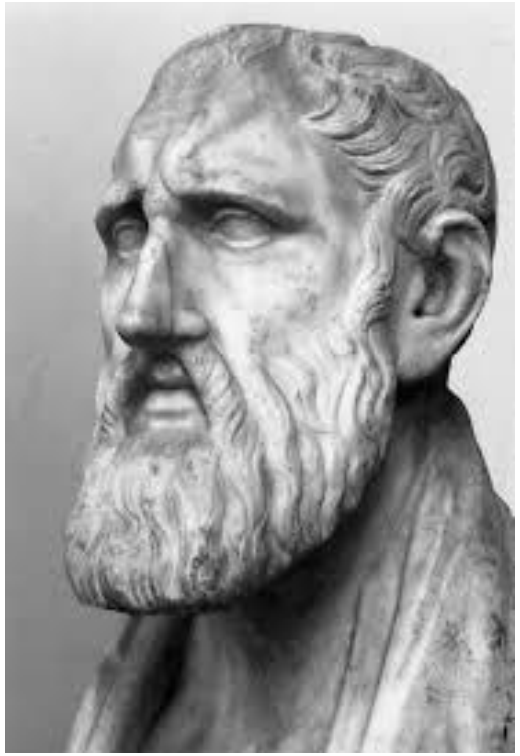
   - Rank-Aware Generalization to Next-Token Prediction



The Capacity for Ranking

Superior Expressive Capacity of ARRs over DEs

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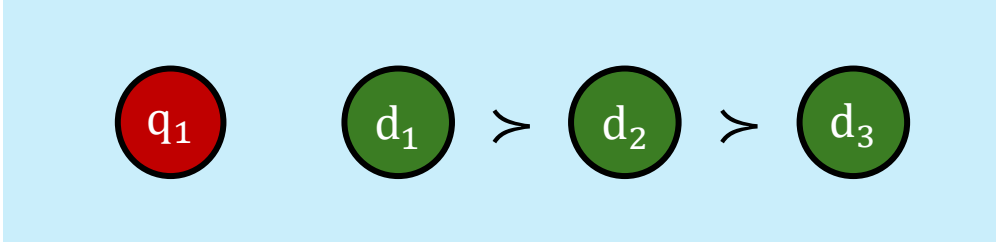
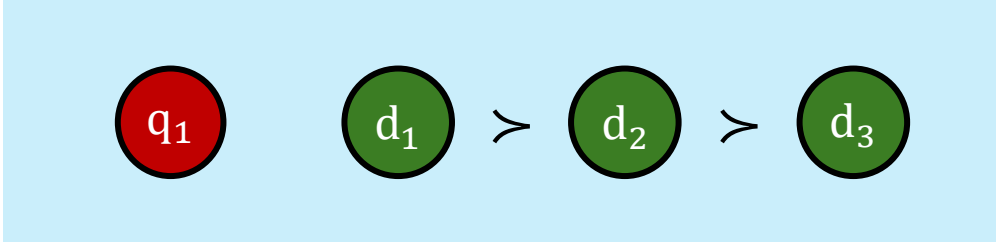


Complete Ranking Task



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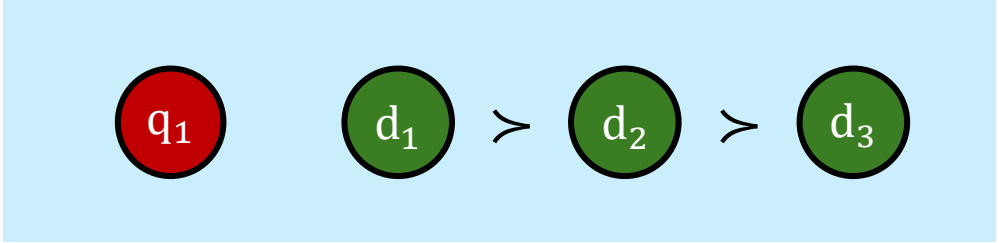

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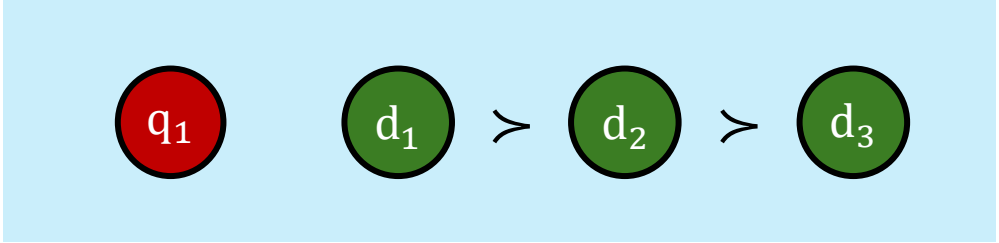

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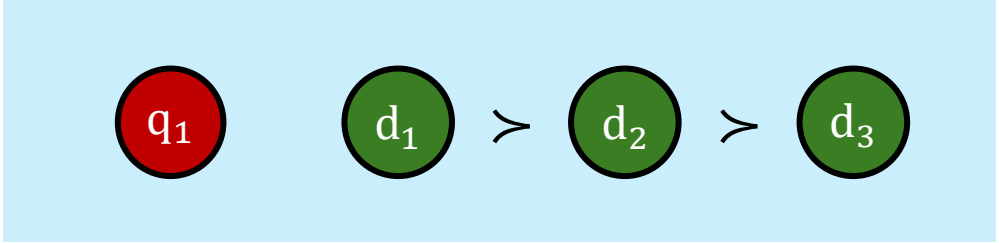

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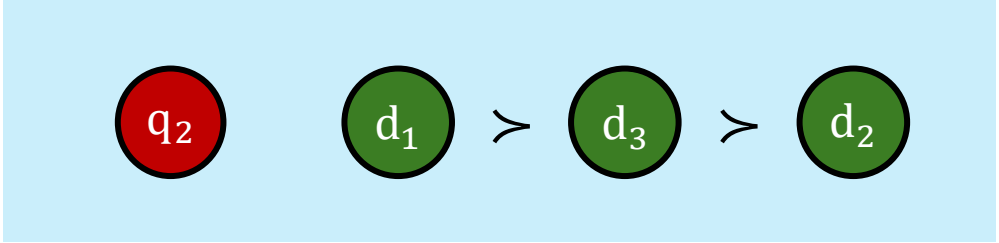

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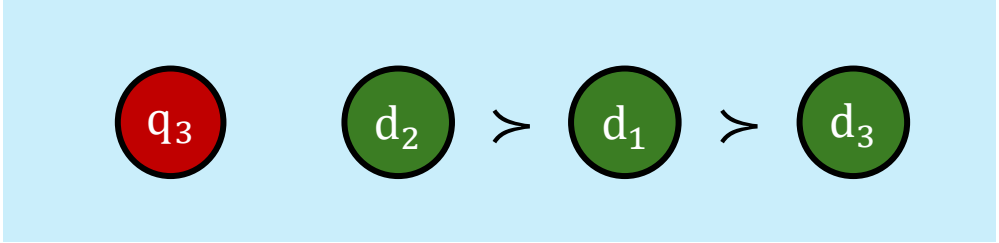

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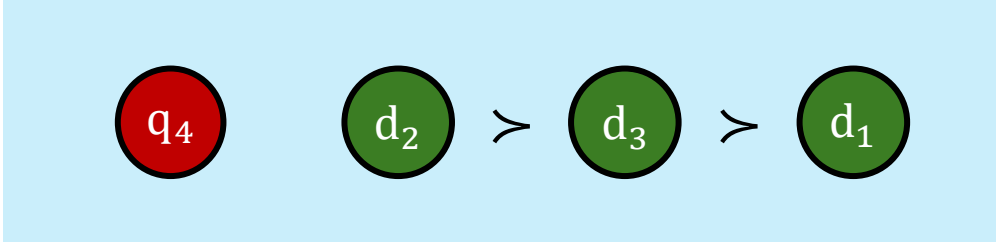

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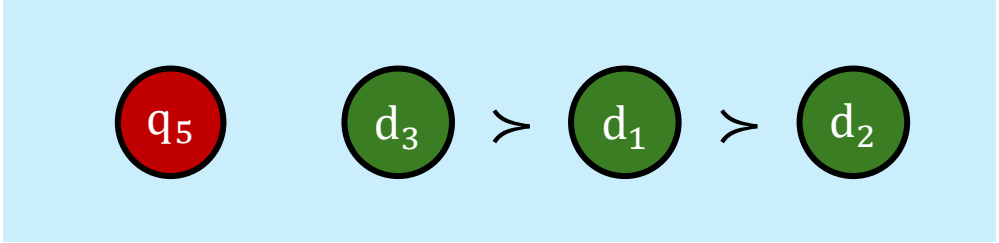

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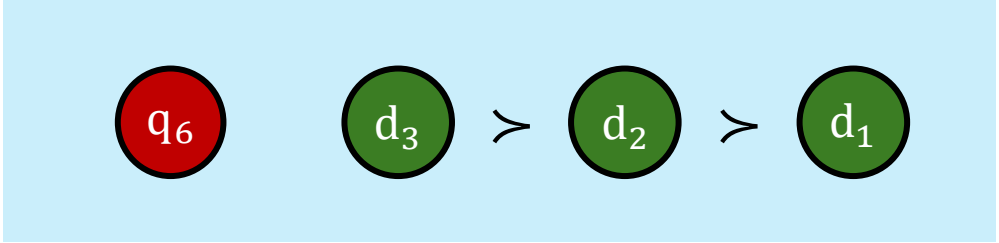

Insufficiency of DEs for Complete Ranking

Let $f_{\mathrm{DE}}(\mathrm{q}, \mathrm{d}; \theta)$ be a dual encoder architecture with embedding dimension n, and let $k = |\mathcal{D}|$. Then there does not exist a θ such that $f_{\mathrm{DE}}(\mathrm{q}, \mathrm{d}; \theta)$ solves a complete ranking task when

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Proof [*Counting Distance Permutations, Skala 2009*]:
Let $N_{n,p}(k)$ denote the maximum number of distinct distance permutations generated by k sites in $\mathbb{R}^n$ with the L_p metric. From Skala 2009, Corollary 8, $N_{n,2}(k) \leq k^{2n}$. When above inequality holds, $k^{2n} < k!$ ■

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By Stirling's formula $\ln(k!) = k \ln k - k + O(\ln(k))$, this becomes $n < \frac{k}{2} - \frac{k}{2 \ln k} + O(1)$, i.e., if $\boldsymbol{n < O(k) = O(|\mathcal{D}|)}$.

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DE embedding dimension

Must grow linearly with corpus size

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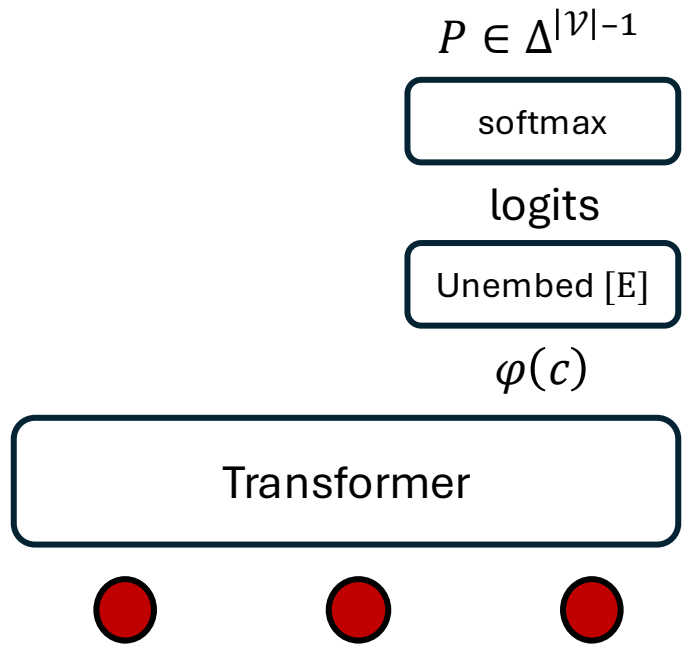

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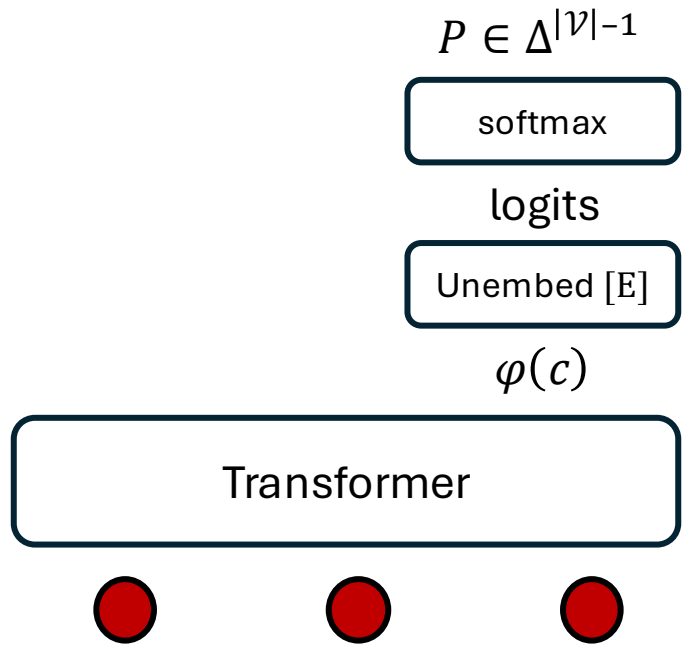

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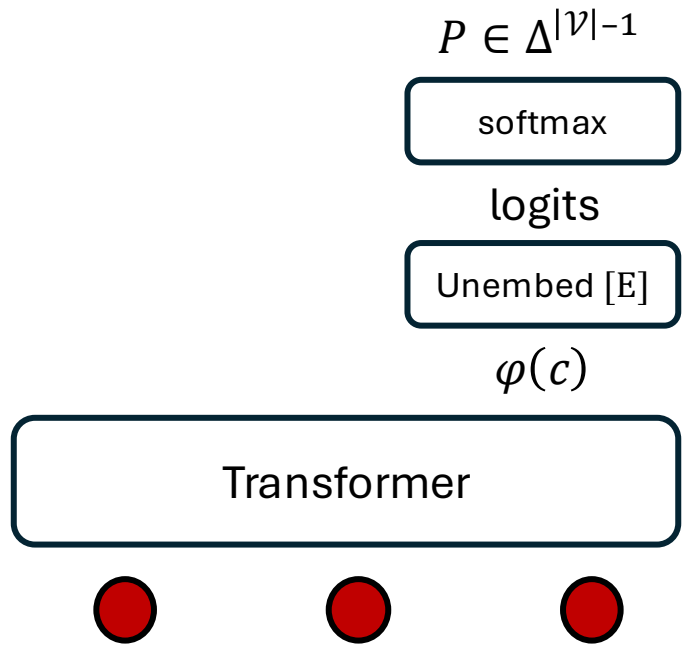

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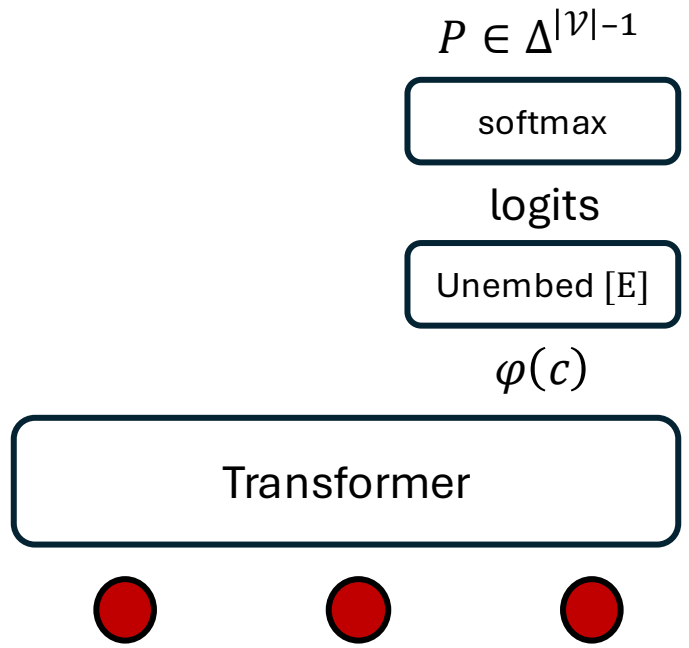

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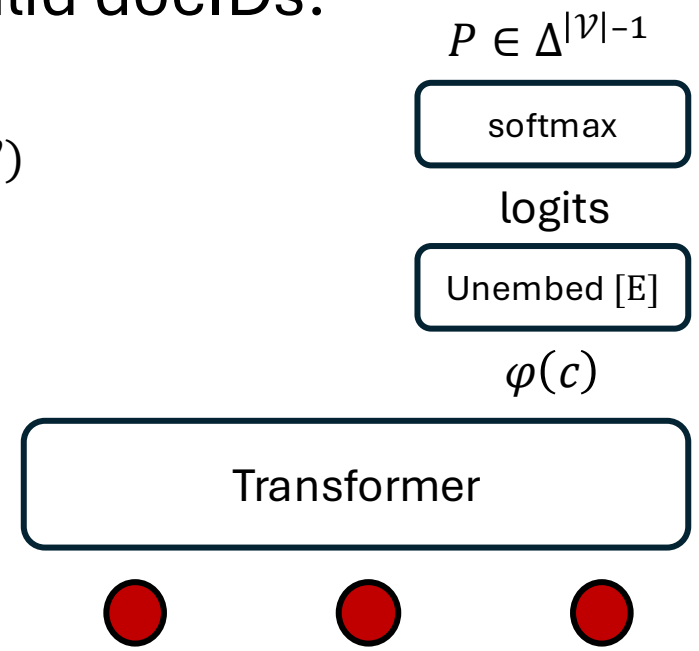
- In practice, use **constrained decoding** to ensure valid docIDs:

$$P(v \mid c) = \begin{cases} \mathrm{softmax}\left(\mathrm{E}_{\mathrm{docIDs}}\varphi(c)\right) & \text{if } v \in \mathcal{V}_{\mathrm{docIDs}} \ \ (\mathcal{V}_{\mathrm{docIDs}} \subset \mathcal{V}) \\ 0 & \text{otherwise} \end{cases}$$



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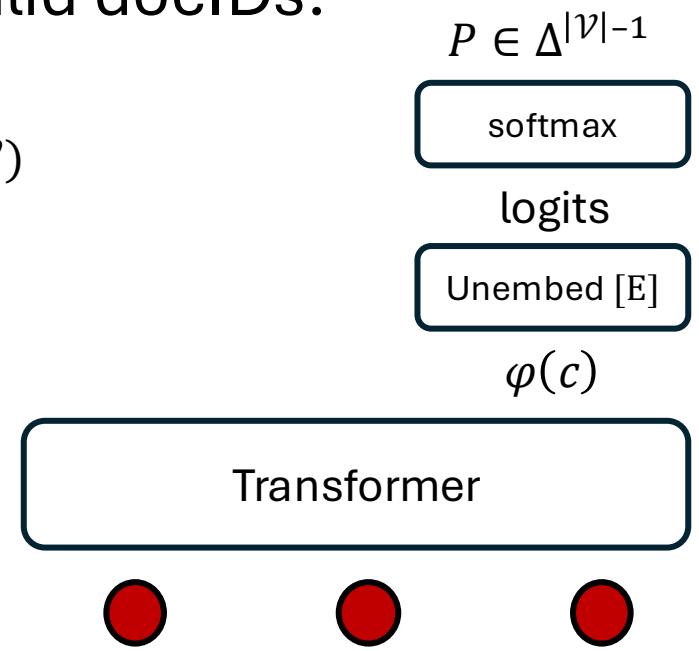
- Define $\mathrm{E}' = [\mathrm{E}_{\mathrm{docIDs}}\ \mathbf{1}_{|\mathcal{V}_{\mathrm{docIDs}}|}]$



ARRs: Arbitrary Distributions over Tokens

An infinite-capacity ARR can produce **any (strictly positive) probability distribution over tokens in $\mathcal{V}_{\text{docIDs}}$** via constrained decoding

$\Leftrightarrow$

$$\text{rank}(\text{E}') = |\mathcal{V}_{\text{docIDs}}|$$

Proof by contradiction. If $\text{rank}(\text{E}') < |\mathcal{V}_{\text{docIDs}}|$*, then we can show that there exists a distribution which can't be hit.*

ARRs: Arbitrary **Distributions over Sequences**

The chain rule of probability implies…

An infinite-capacity ARR can produce **any (strictly positive) probability distribution over sequences in** $\mathcal{V}_{\text{docIDs}}$ via constrained decoding

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ARRs: Arbitrary **Permutations over Tokens**

*Clearly, arbitrary distributions over sequences implies arbitrary rankings over sequences. But is the rank requirement also **necessary** for arbitrary rankings over sequences?*

An infinite-capacity ARR can produce **any (strictly positive) permutations over tokens in $\mathcal{V}_{\text{docIDs}}$** via constrained decoding

$\Leftrightarrow$

$$\text{rank}(\text{E}') = |\mathcal{V}_{\text{docIDs}}|$$

Rank requirement is both **necessary and sufficient** to enable a fixed-dimension ARR to generate arbitrary probability distributions or arbitrary permutations for an unrestricted number of docIDs.

Practical Implications

- Is theoretical requirement on E' satisfied by LLMs used for reranking?
- Why are LLMs successful even if requirement can't be satisfied?
- WordNet
  - docIDs are English nouns $\Rightarrow |\mathcal{V}_{\mathrm{docIDs}}| \approx |\mathcal{V}|$.
  - $n \ll |\mathcal{V}| \Rightarrow \mathrm{rank}(\mathrm{E}') \ll |\mathcal{V}_{\mathrm{docIDs}}|$
  - `Mistral-7B-v0.3-it` has $|\mathcal{V}| = 32{,}768$ and $d = 4{,}096$
  - Most possible sequences never encountered; model needs to learn tiny subset of all possible permutations.
- ESCI Shopping Queries
  - we choose numerical docIDs $\Rightarrow n \approx |\mathcal{V}_{\mathrm{docIDs}}|$
  - rank requirement on E' can be matched more accurately

A Rank-Aware Loss Function

Incorporating Rank-Awareness into Next Token Prediction

STOICAL: **S**imple **To**ken-**I**tem **Ca**librated **L**oss

$$\mathcal{L}(q, d_r, r; \theta)$$

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model-generated per-timestep probabilities

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Prefix Tree (Trie) Construction

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q: “a common pet”

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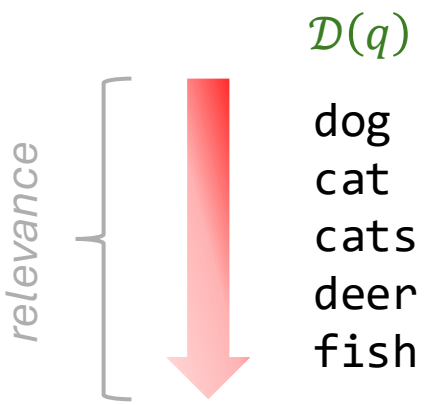

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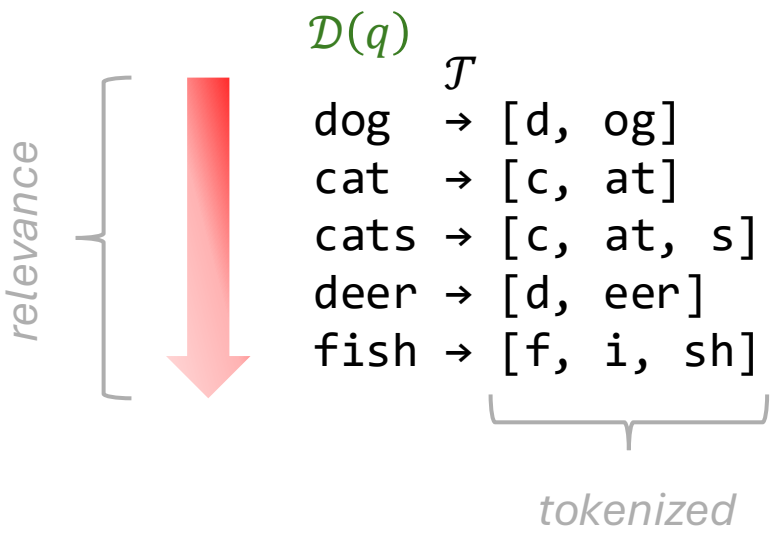

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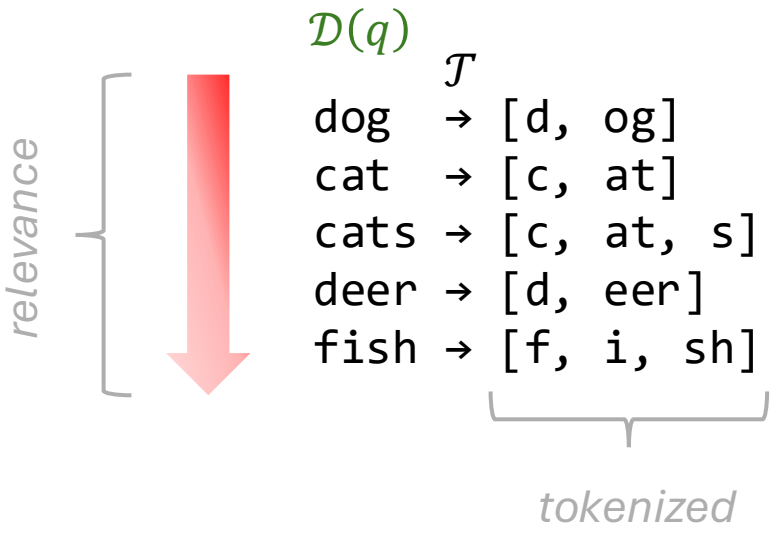

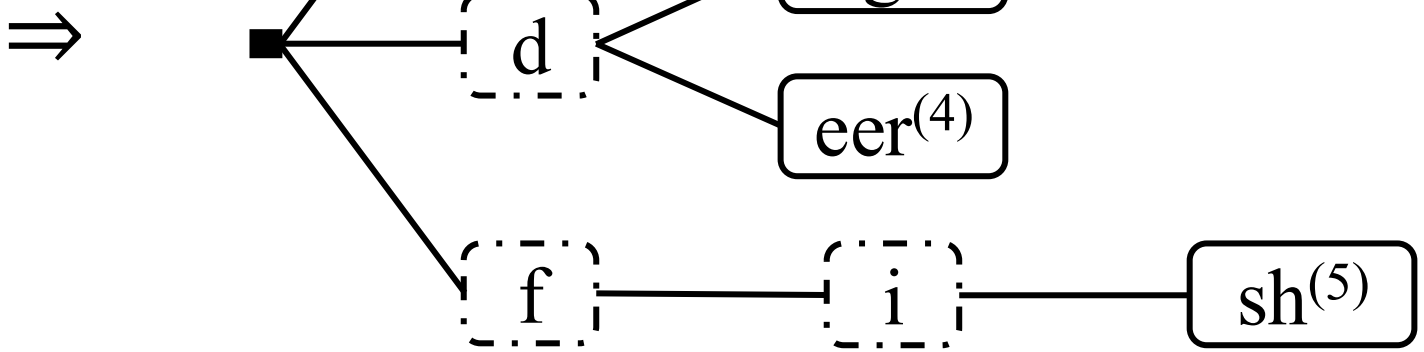

Teacher Forcing with One Hot Supervision

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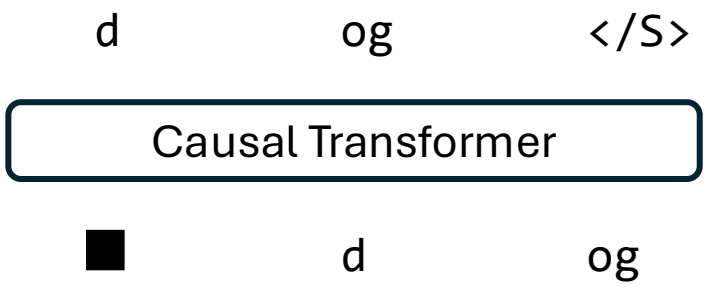

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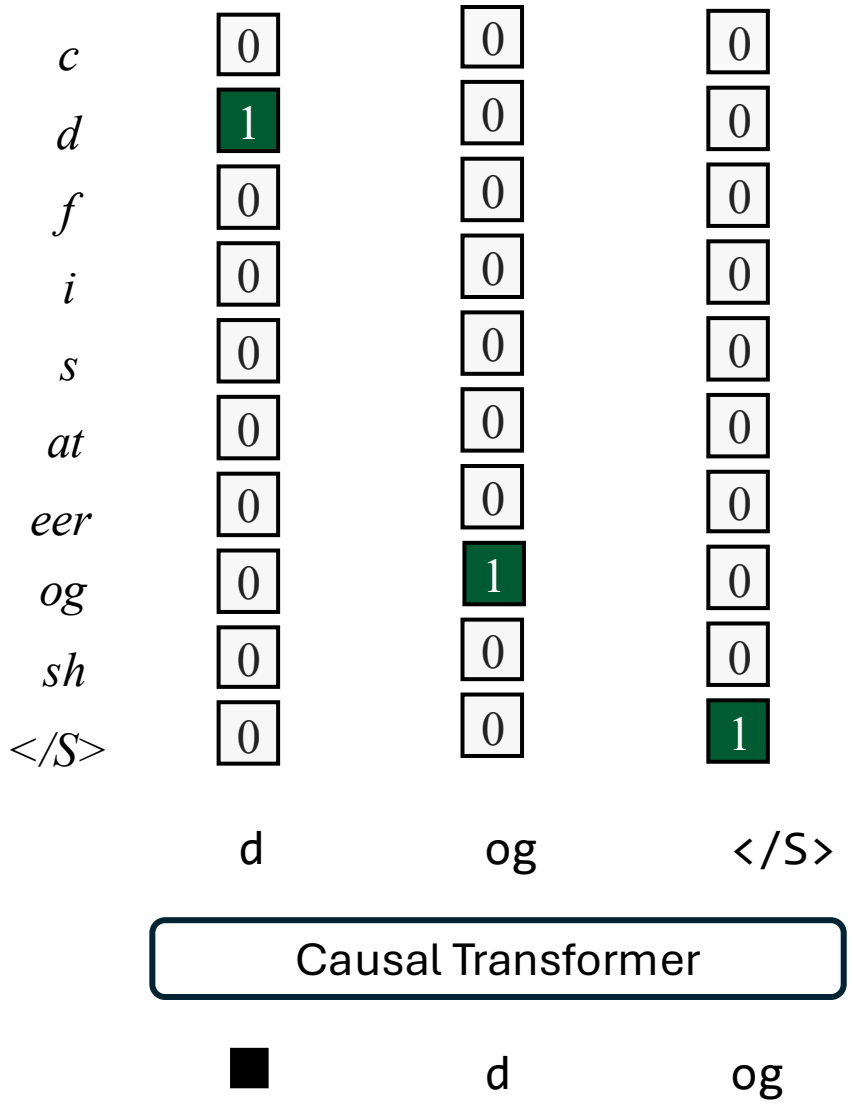

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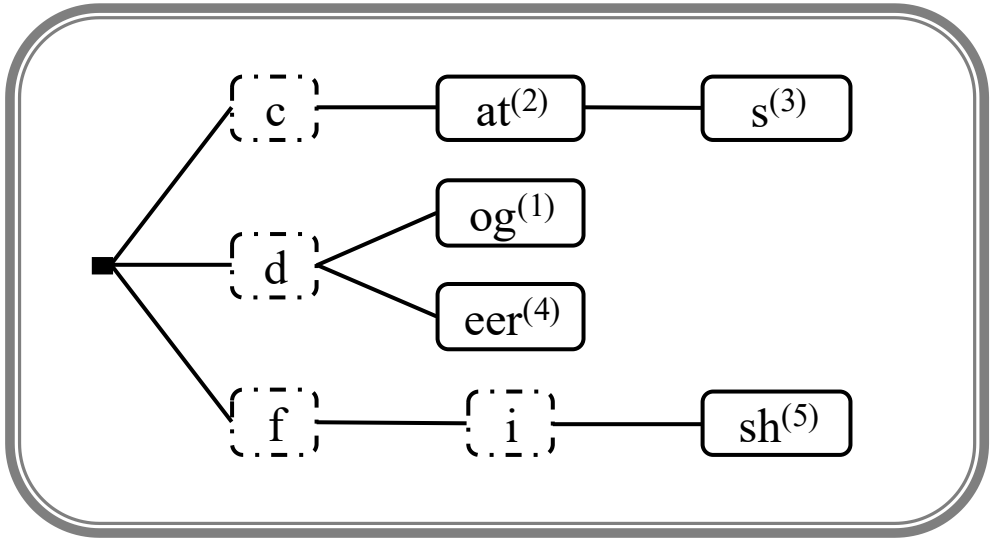

Causal Transformer

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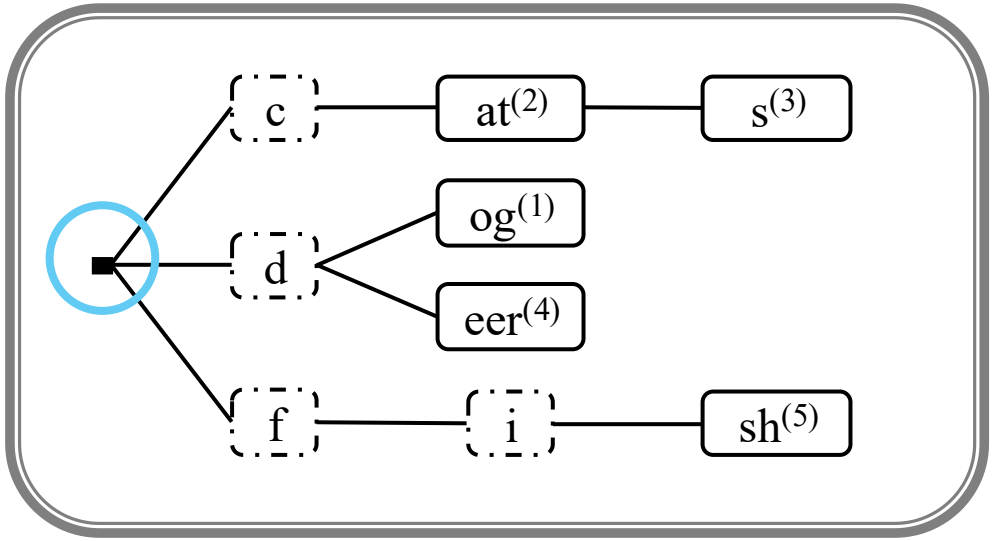

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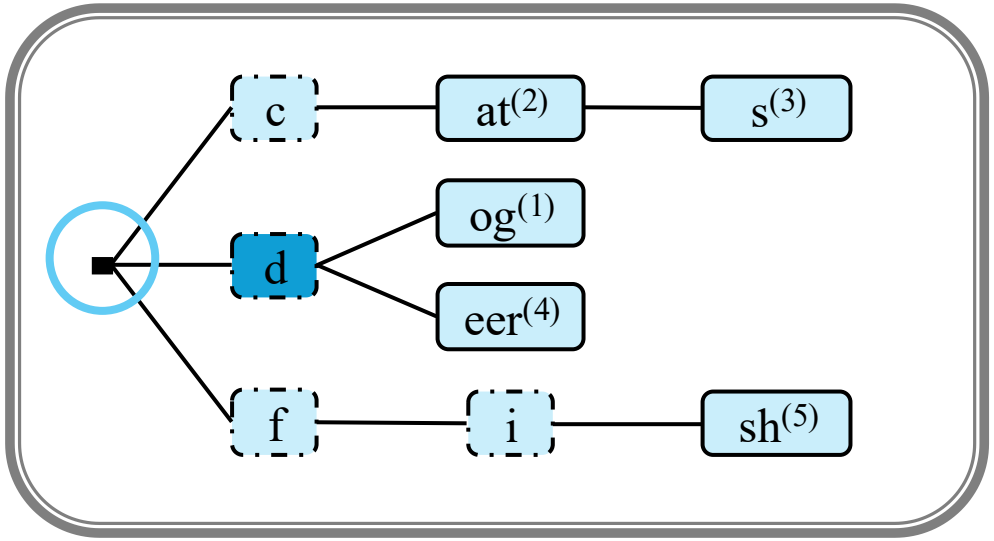

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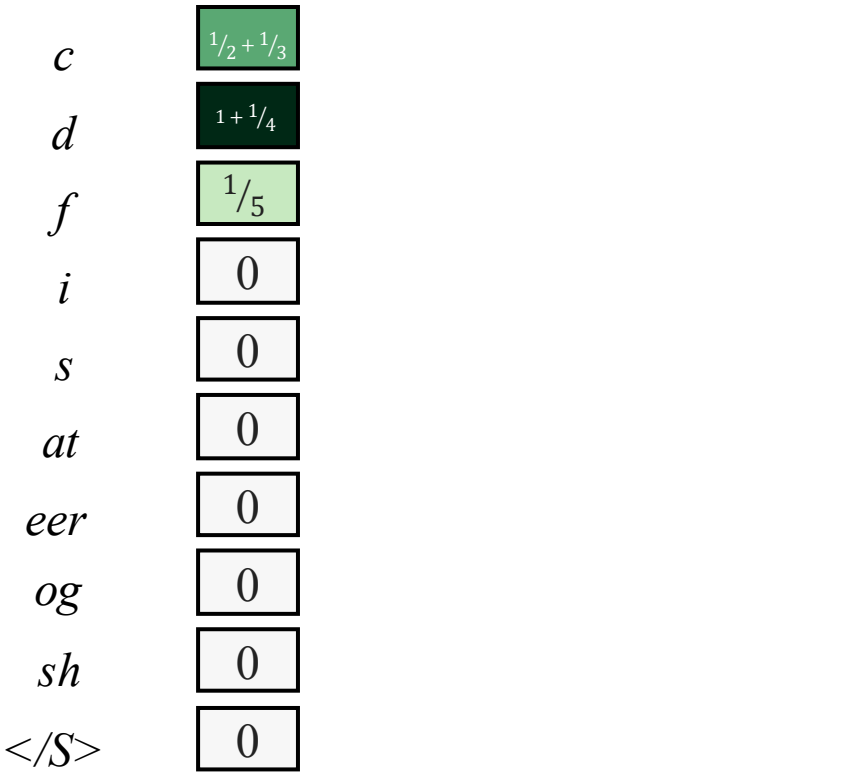

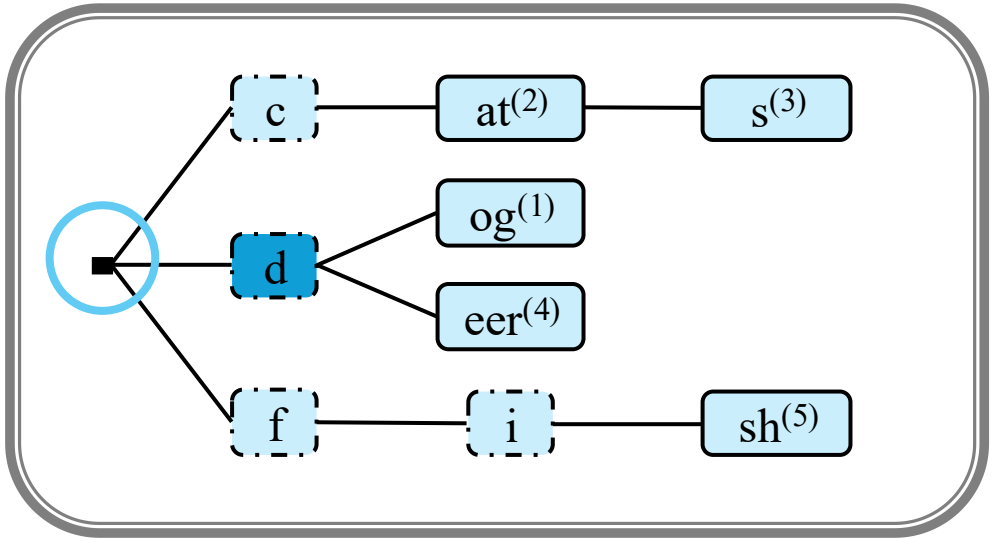

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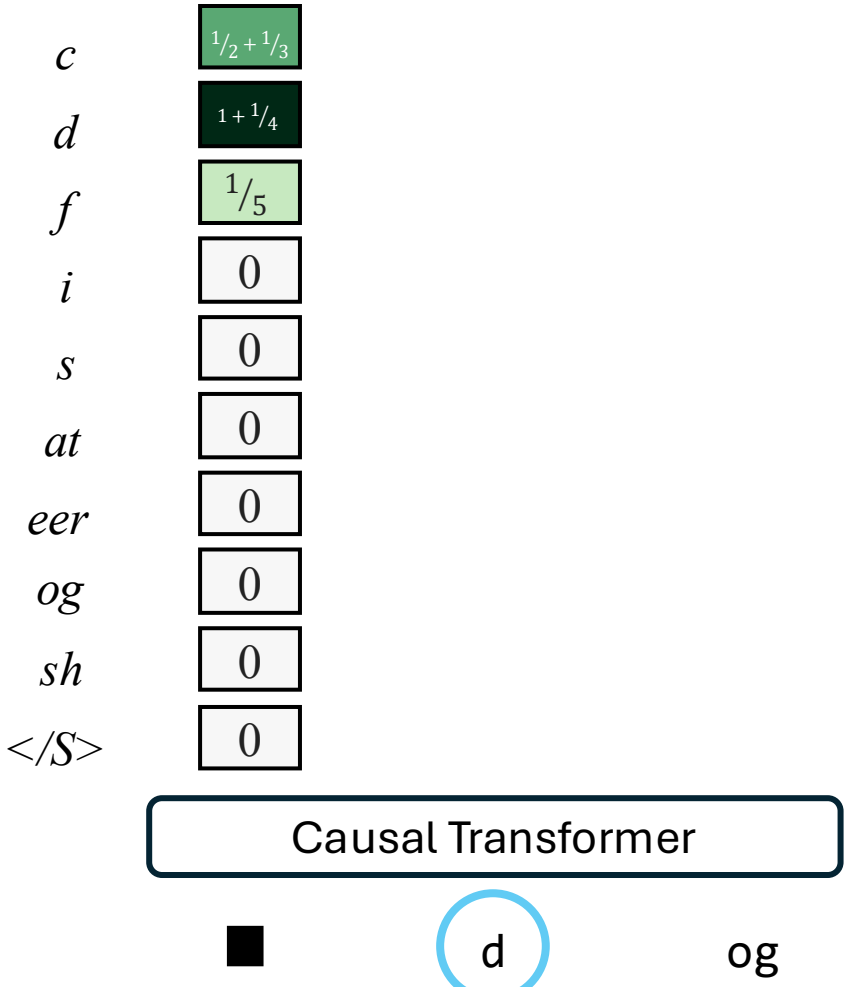

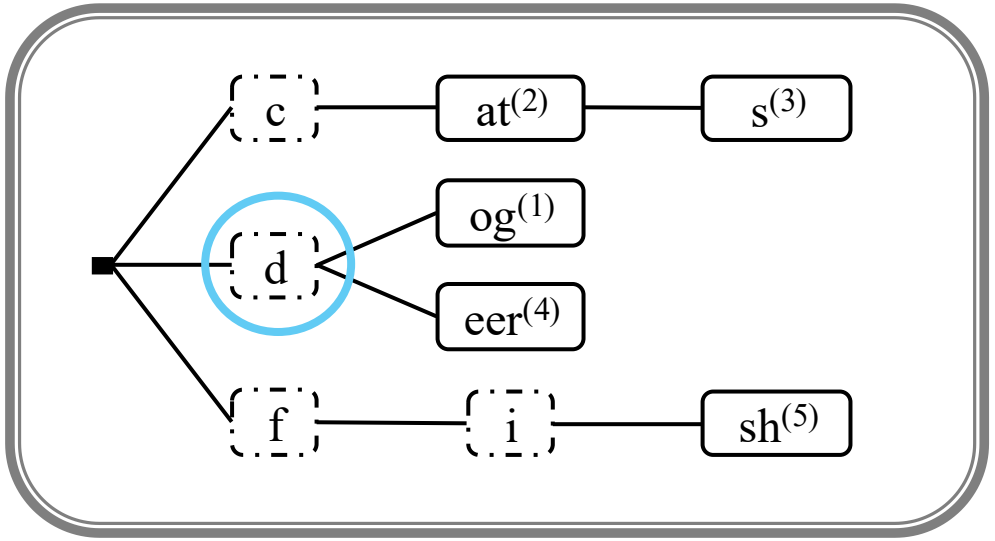

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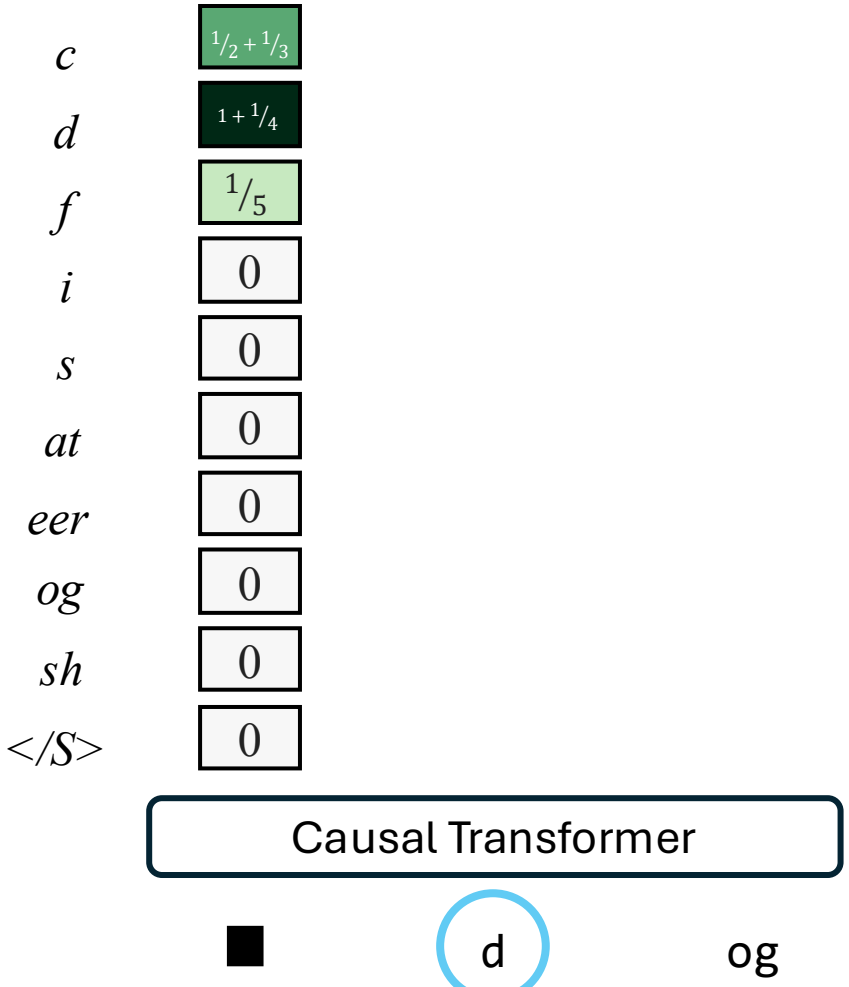

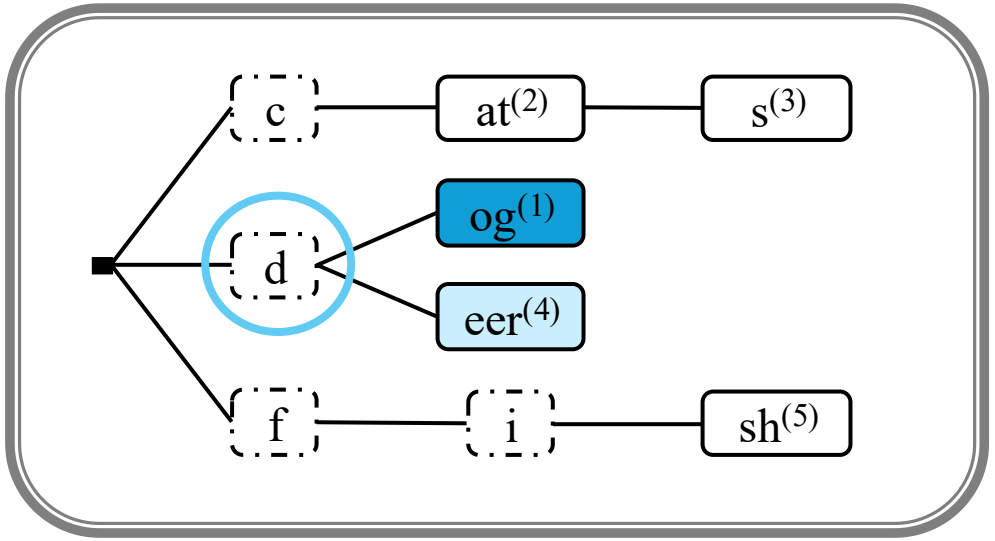

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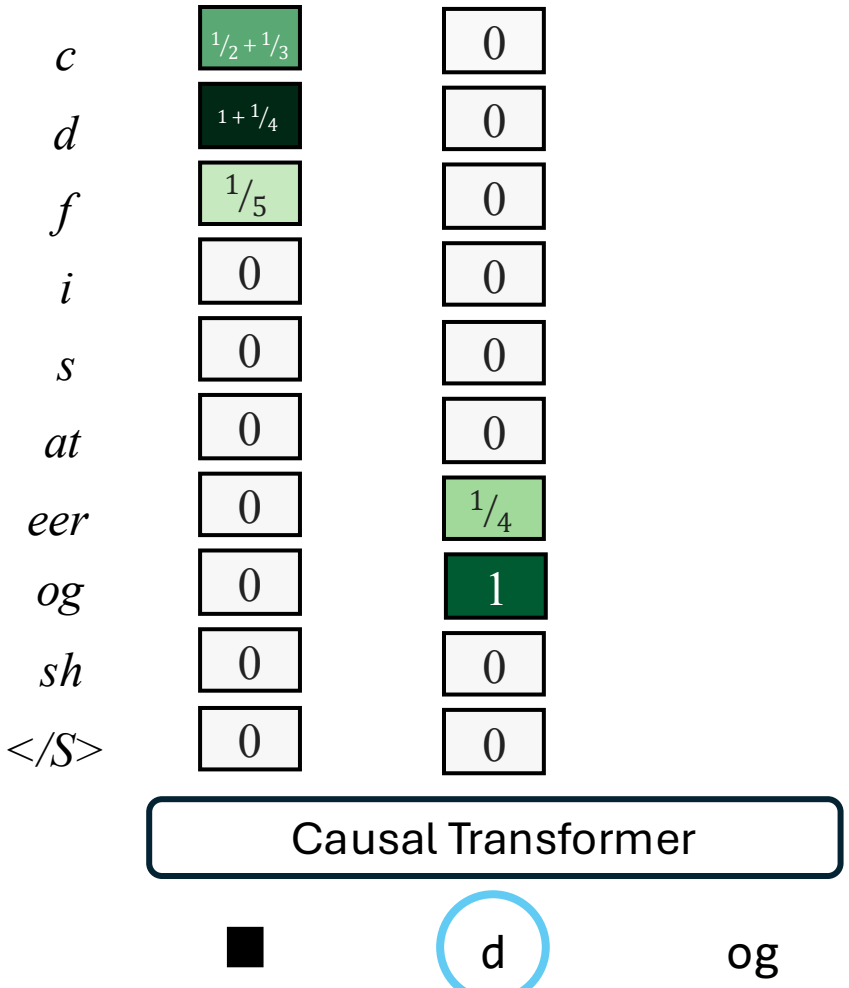

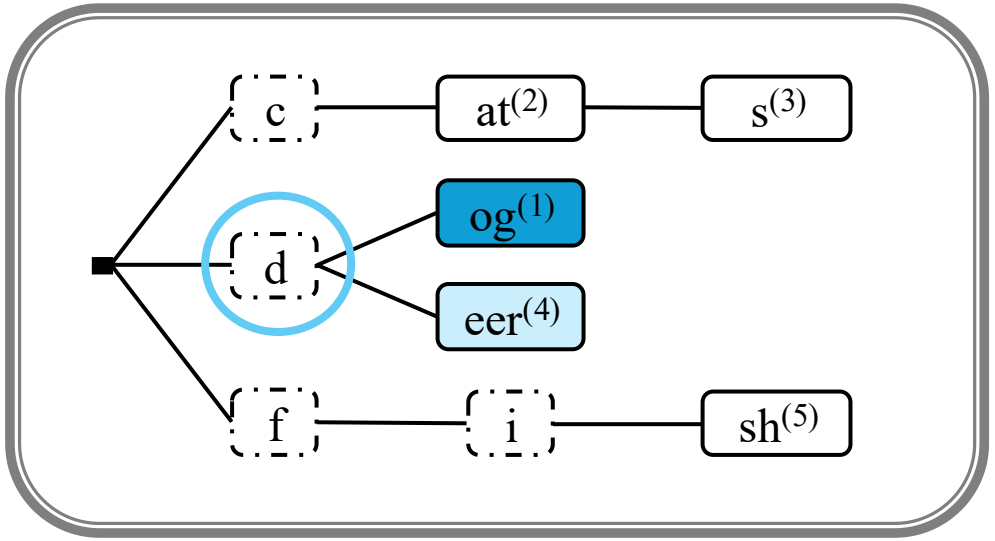

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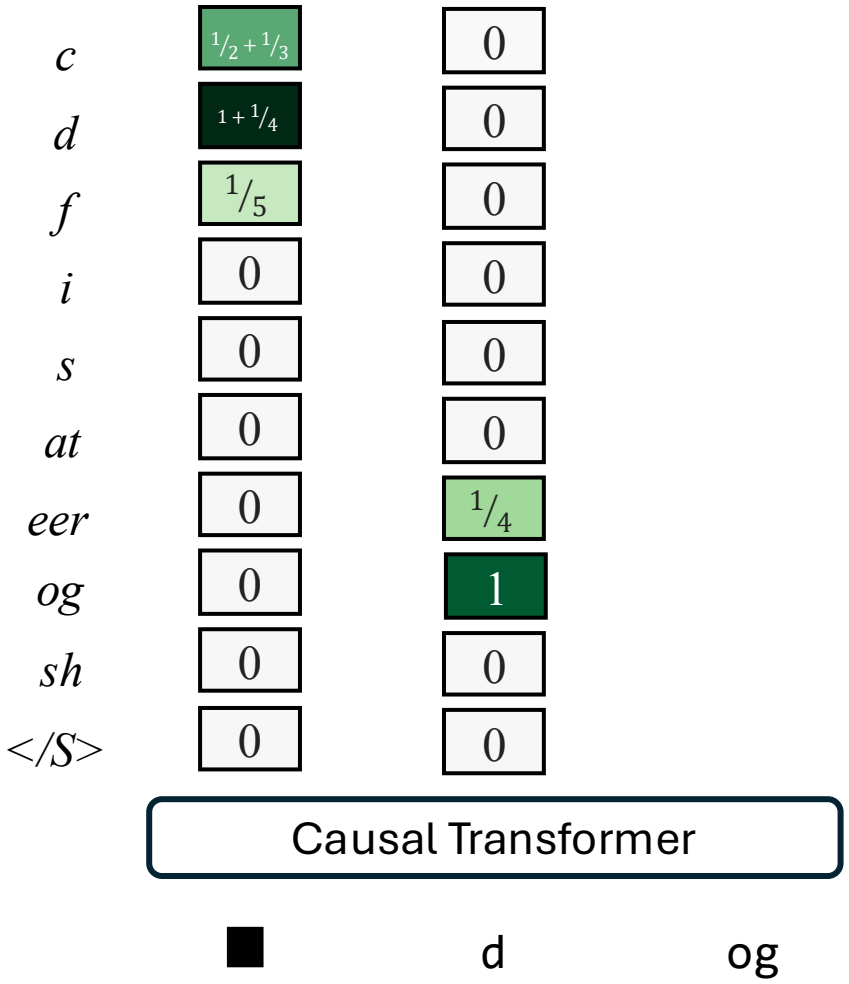

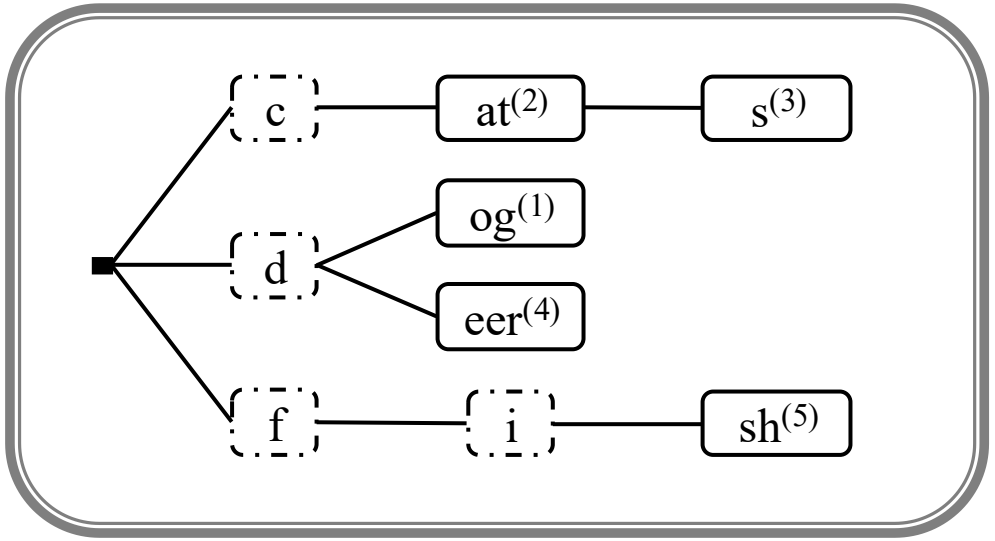

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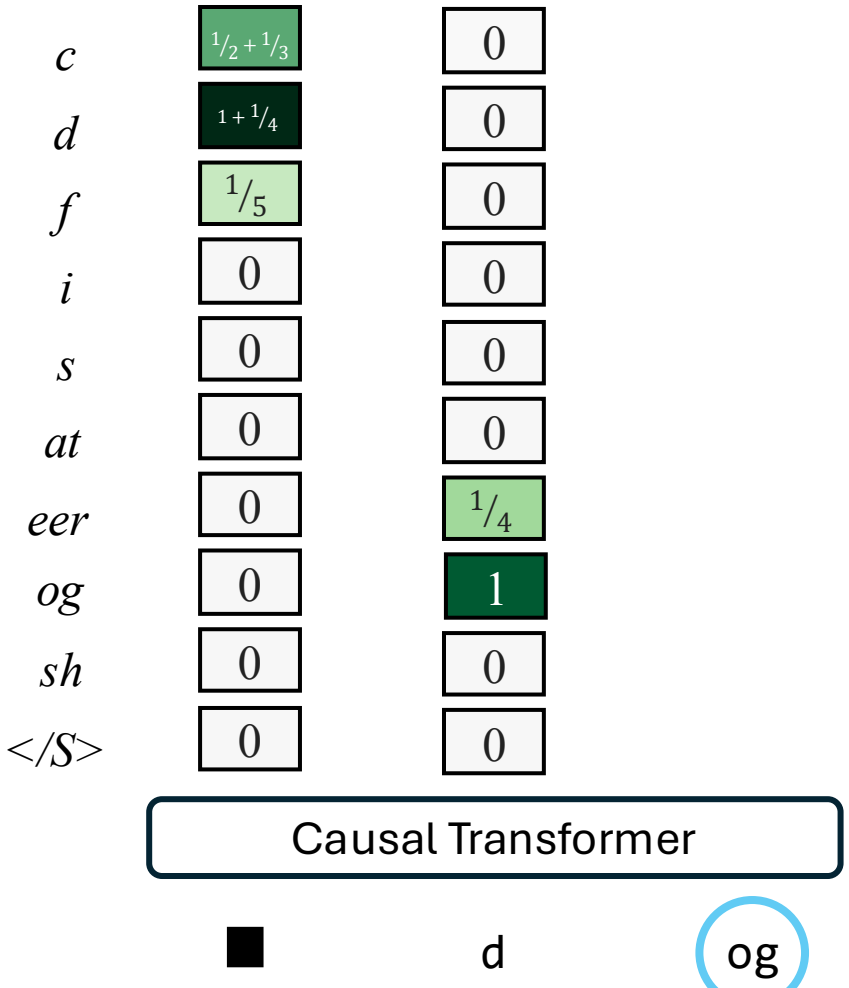

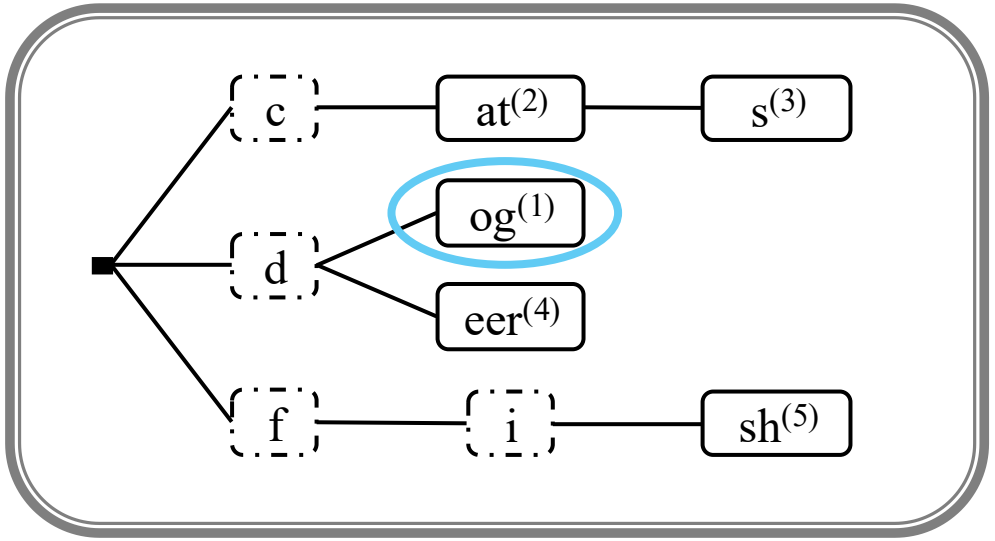

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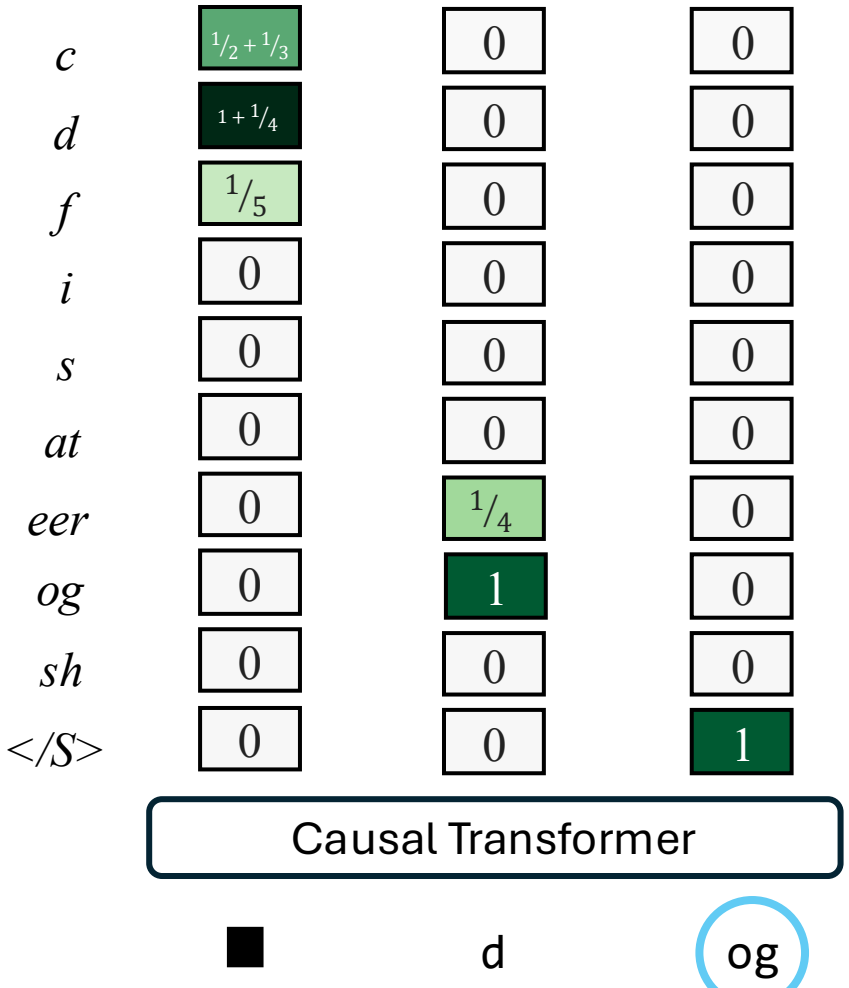

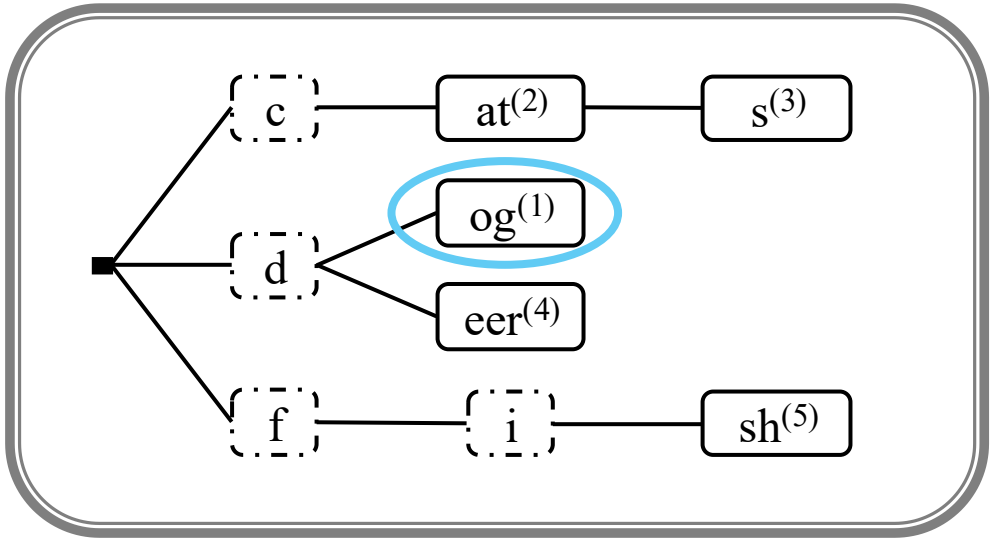

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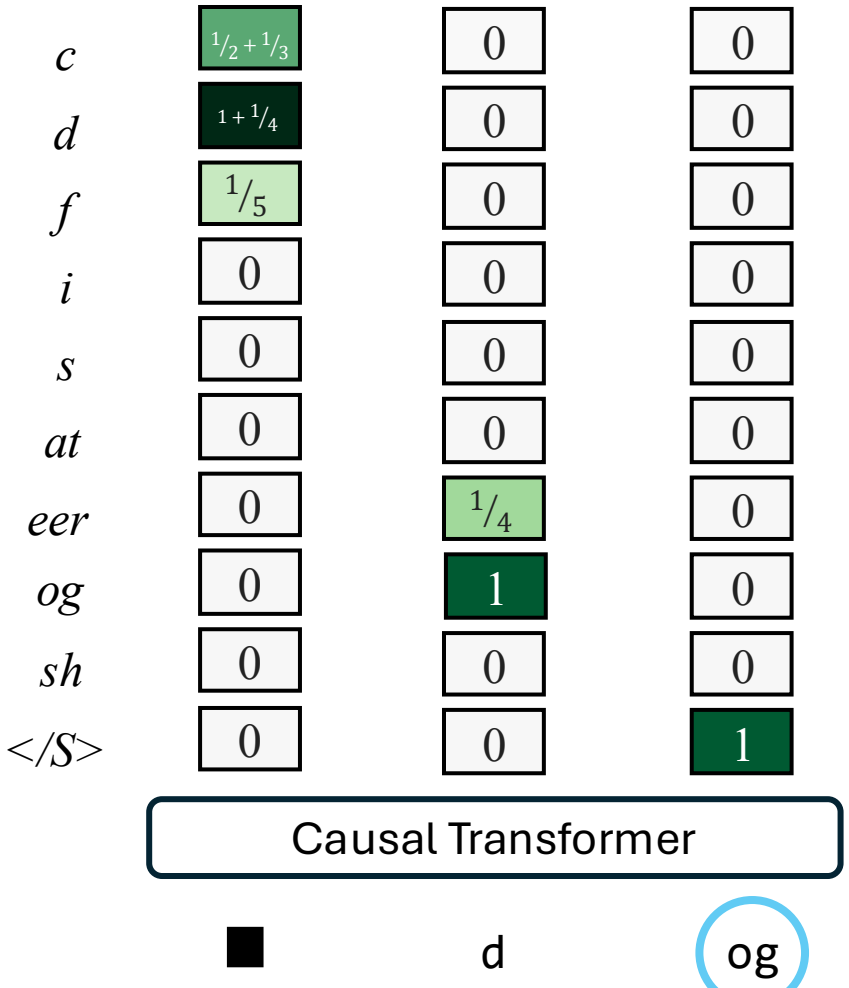

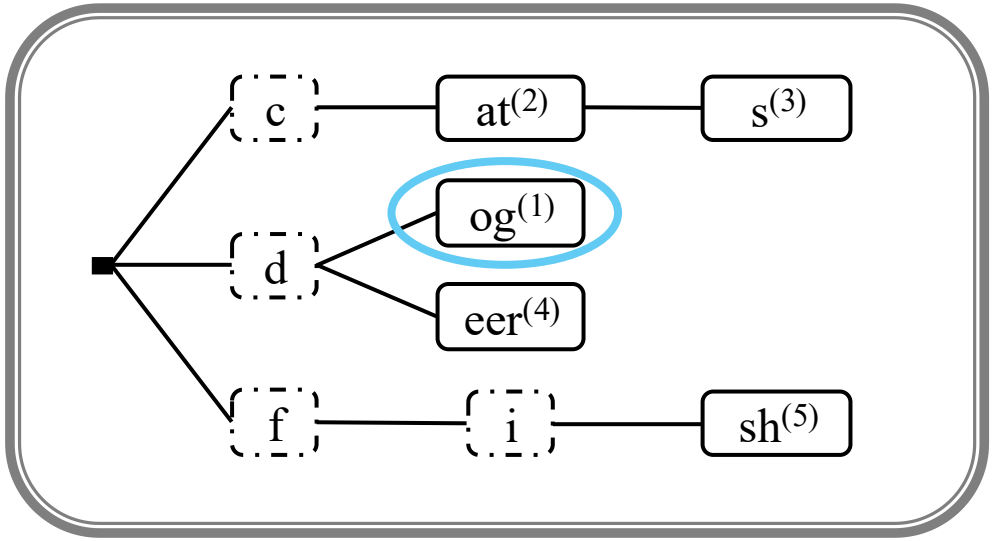

** Leaf weights can be controlled by temperature β*

Datasets & DocID Creation

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- WordNet
  - Query: *deer*
  - Ranked docIDs: *ruminant ≻ even-toed ungulate ≻ ungulate ≻ placental ≻ mammal ≻ vertebrate ≻ chordate ≻ animal ≻ organism ≻ living thing ≻ whole ≻ object ≻ physical entity ≻ entity*
  - Negative docID: *solarization*

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- ESCI Shopping Queries
  - Embed queries and product titles with `gecko-1b-en`
  - For each query, take 1st, 100th, …, 10,000th closest product titles
  - Sparse dictionary learning → vectors in $\mathbb{R}^{100}$ with 3 nonzero entries
  - Indices sorted in descending order by absval of entries, e.g., “25, 36, 39”

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  - $\text{score}(d_1^+) = \log(n_q + 1), \text{score}(d_2^+) = \log(n_q) \ldots, \text{score}\left(d_{n_q-1}^+\right) = \log(3), \text{score}\left(d_{n_q}^+\right) = \log(2), \text{score}(d_q^-) = \log(1) = 0$

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Results: WordNet

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NTP: $\lambda(r) = \mathbb{I}_{r=1}$ **y** one-hot		27.66%	94.89	**99.96**	62.43	55.3	55.16	63.42
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Table 1 | WordNet results (evaluated over 5000 (query, targets)-examples)

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Results: ESCI

- *ESCI too big to train with item-level reweighting for each item*

		nDCG (↑)	R@1 (↑)	R@2 (↑)	R@5 (↑)	R@10 (↑)	R@25 (↑)	R@50 (↑)
NTP: $\lambda(r) = \mathbb{I}_{r=1}$ **y** one-hot		95.23	**95.16**	52.58	27.64	23.51	37.98	62.99
$\lambda(r) = \mathbb{I}_{r=1}$ **y** marg. over trie w/ leaf scores $\frac{1}{r^\beta}$	$\beta = 1$	**97.21**	70.0	56.61	45.57	46.27	54.78	**69.58**
	$\beta = 2$	**97.21**	70.32	**58.06**	**51.07**	**48.08**	**54.96**	69.03
	$\beta = 3$	97.11	68.71	56.13	48.03	45.78	52.31	67.63
	$\beta = 4$	96.96	69.03	54.84	44.6	43.9	50.29	67.7
	$\beta = 5$	96.89	66.77	56.13	44.21	42.4	50.72	67.61

Table 2 | ESCI results (evaluated over 310 (query, targets)-examples)

Results: ESCI

- *ESCI too big to train with item-level reweighting for each item*

		nDCG (↑)	R@1 (↑)	R@2 (↑)	R@5 (↑)	R@10 (↑)	R@25 (↑)	R@50 (↑)
NTP: $\lambda(r) = \mathbb{I}_{r=1}$ **y** one-hot		95.23	**95.16**	52.58	27.64	23.51	37.98	62.99
$\lambda(r) = \mathbb{I}_{r=1}$ **y** marg. over trie w/ leaf scores $\frac{1}{r^\beta}$	$\beta = 1$	**97.21**	70.0	56.61	45.57	46.27	54.78	**69.58**
	$\beta = 2$	**97.21**	70.32	**58.06**	**51.07**	**48.08**	**54.96**	69.03
	$\beta = 3$	97.11	68.71	56.13	48.03	45.78	52.31	67.63
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	$\beta = 5$	96.89	66.77	56.13	44.21	42.4	50.72	67.61

Table 2 | ESCI results (evaluated over 310 (query, targets)-examples)

- *Trie-based marginalization economical alternative*

Results: ESCI

- *ESCI too big to train with item-level reweighting for each item*

		nDCG (↑)	R@1 (↑)	R@2 (↑)	R@5 (↑)	R@10 (↑)	R@25 (↑)	R@50 (↑)
NTP: $\lambda(r) = \mathbb{I}_{r=1}$ **y** one-hot		95.23	**95.16**	52.58	27.64	23.51	37.98	62.99
$\lambda(r) = \mathbb{I}_{r=1}$ **y** marg. over trie w/ leaf scores $\frac{1}{r^\beta}$	$\beta = 1$	**97.21**	70.0	56.61	45.57	46.27	54.78	**69.58**
	$\beta = 2$	**97.21**	70.32	**58.06**	**51.07**	**48.08**	**54.96**	69.03
	$\beta = 3$	97.11	68.71	56.13	48.03	45.78	52.31	67.63
	$\beta = 4$	96.96	69.03	54.84	44.6	43.9	50.29	67.7
	$\beta = 5$	96.89	66.77	56.13	44.21	42.4	50.72	67.61

Table 2 | ESCI results (evaluated over 310 (query, targets)-examples)

- *Trie-based marginalization economical alternative*
- *Synergy between token-level loss and trie-friendly docIDs*

CE and DE settings

CE and DE settings

- *Lookup table DEs with weighted batch softmax loss:*

$$\mathcal{L}_{\mathrm{DE}}(\{q_i, d_i, r_i\}_{i=1}^{B}; \phi) = -\frac{1}{B}\sum_{i=1}^{B}\left(\lambda(r_i)\cdot\log\frac{\exp\left(\langle \mathrm{E}_\phi(q_i), \mathrm{E}_\phi(d_i)\rangle/\tau\right)}{\sum_{j=1}^{B}\exp\left(\langle \mathrm{E}_\phi(q_i), \mathrm{E}_\phi(d_j)\rangle/\tau\right)}\right)$$

CE and DE settings

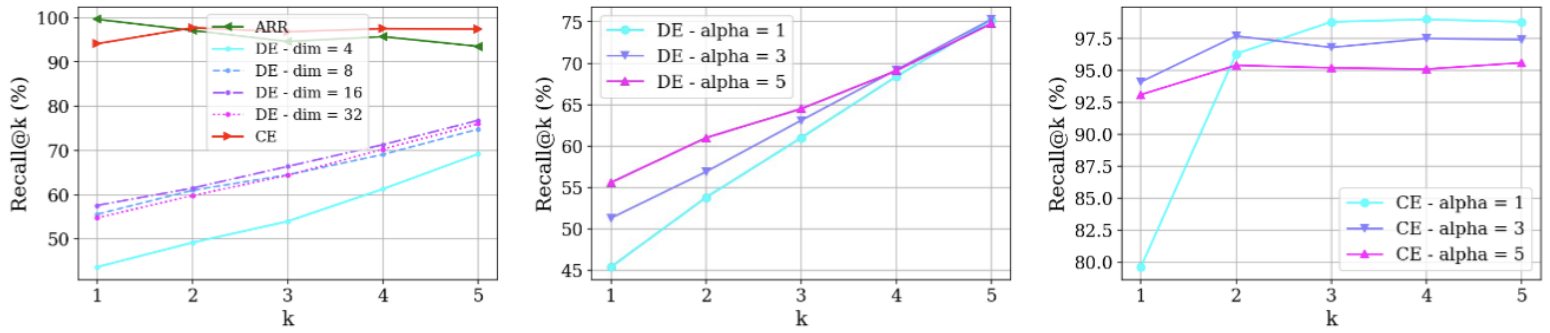
- *Lookup table DEs with weighted batch softmax loss:*

$$\mathcal{L}_{\mathrm{DE}}(\{q_i, d_i, r_i\}_{i=1}^{B}; \phi) = -\frac{1}{B}\sum_{i=1}^{B}\left(\lambda(r_i)\cdot\log\frac{\exp\left(\langle \mathrm{E}_\phi(q_i), \mathrm{E}_\phi(d_i)\rangle/\tau\right)}{\sum_{j=1}^{B}\exp\left(\langle \mathrm{E}_\phi(q_i), \mathrm{E}_\phi(d_j)\rangle/\tau\right)}\right)$$

- *CEs with randomly chosen permutation for negative set:*

$$\mathcal{L}_{\mathrm{CE}}(\mathcal{P}, \mathcal{N}; \phi, \psi) = -\sum_{i=1}^{B}\left(\frac{\lambda(r_i)}{\sum_{j=1}^{B}\lambda(r_j)}\log\sigma\Big(f_{\mathrm{MLP}}(\mathrm{concat}(\mathrm{E}_\phi(q_i), \mathrm{E}_\phi(d_i)); \psi)\Big) + \frac{1}{B}\log\sigma\Big(-f_{\mathrm{MLP}}(\mathrm{concat}(\mathrm{E}_\phi(q_i), \mathrm{E}_\phi(d_{\rho(i)})); \psi)\Big)\right)$$

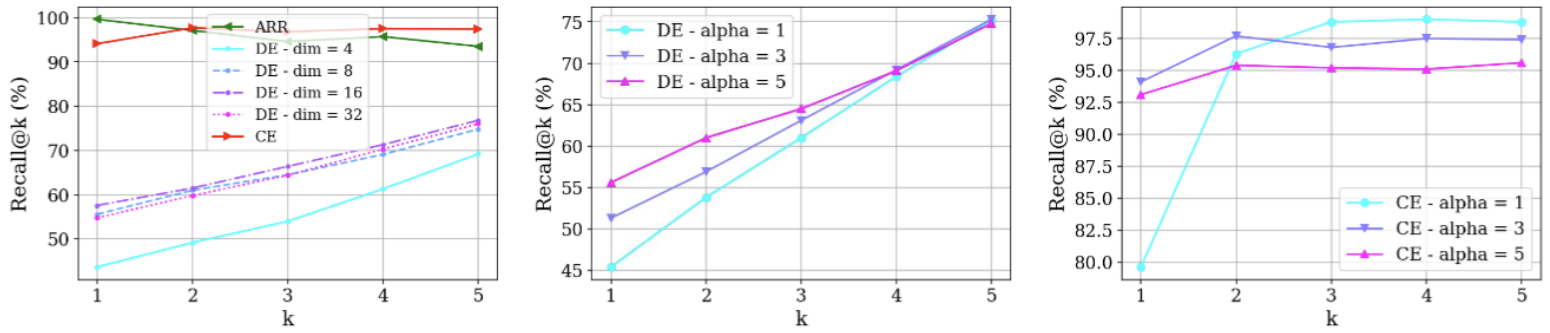
Comparison with DEs and CEs



(a) Comparison with DE/CE (b) Effect of α for DE (b) Effect of α for CE

Figure 3 | Results on WordNet with Dual Encoders (DEs) and Cross Encoders (CEs). (a) Comparison of autoregressive ranking with DEs ($n \in \{4, 8, 16, 32\}, \alpha = 5$) and CE ($n = 32$). (b, c) Effect of α in the reweighting function for DEs ($n = 8$) and CEs ($n = 32$), respectively.

Comparison with DEs and CEs



(a) Comparison with DE/CE (b) Effect of α for DE (b) Effect of α for CE

Figure 3 | Results on WordNet with Dual Encoders (DEs) and Cross Encoders (CEs). (a) Comparison of autoregressive ranking with DEs ($n \in \{4, 8, 16, 32\}, \alpha = 5$) and CE ($n = 32$). (b, c) Effect of α in the reweighting function for DEs ($n = 8$) and CEs ($n = 32$), respectively.

- *ARRs and CEs able to saturate dataset whereas DEs cannot*

Thank you!